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NUMERICAL SIMULATION OF A PETROLEUM RESERVOIR: A CASE STUDY OF WATER INJECTION BY THE APPLICATION OF ISOTHERMAL AND THERMAL METHODS FOR HEAVY OILS

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ABSTRACT

Petroleum industry has put efforts on developing technologies capable of increasing oil recovery and taking maximum potential of reservoirs. The current paper proposes a comparison of water injection scenarios in a reservoir of dead oil with homogenous, isotropic, and no failure porous medium. Four case studies were set up with specific temperatures and densities for water and reservoir to provide information about the main parameters related to conventional oil recovery methods. The scheme injection is staggered lines, with an injection well centered on the reservoir and four production wells around it. The simulations were run on ANSYS CFX software and results demonstrated an improvement of 31.26% on behalf of the thermal method for an 11 °API oil-type and 10.58% of improvement for the thermal method and a 17 °API oil-type. Therefore, thermal methods proved to be more efficient for heavy oils than for light oils.

KEYWORDS

conventional oil recovery; hot water injection; heavy oil; computer fluid dynamics; numerical simulation

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1. INTRODUCTION

The process of injecting water is already a well-established and useful method to improve oil recovery. According to **Rosa et al. (2011)**, the most used conventional secondary recovery method in the world is the water injection, which was first used in the Bradford field (Pennsylvania, US) in the beginning of the twentieth century.

Oil recovery by fluid injection is a process in which part of the energy used for moving oil through the reservoir is derived from extraneous sources **(Torrey, 1951)**. From different processes of water injection, it is possible to keep the reservoir pressure constant or at least to reduce pressure loss and keep the oil production. The injected water sweeps reservoir porous medium and moves mobile oil to points of less pressure in direction to the production wells. The efficiency of injection process will rely on some parameters such as mobility ratio, injectivity, permeability, porosity, and fluid properties. It is possible to simulate these processes numerically by computational simulation and represent the phenomenon precisely. The behavior of this model is based on three basic equations from the fluid mechanics and reservoir engineering: continuity equation, momentum equation, and Darcy's law. For heat transfer cases, the energy equation is used for balancing energy that is being transferred by the heat across the reservoir. Together, these equations are fundamental to predict oil production and water-oil flowing pattern.

Any kind of injection process will face oil flowing resistance represented by oil viscosity. Some experiences have showed how influential this parameter can be during the process of fluid injection. **Zhao and Turta (2004)** simulated different schemes of injection and proved how high oil viscosity affects oil recovery factor even when the relevant term is not mobility ratio. Rather, it is led by the gravity segregation effect as it is seen in the Toe-To-Heel Waterflooding (TTHW) method. Using both Vertical to Vertical (VTV) method- conventional way of injecting water and producing oil with all wells being vertical- and TTHW method, as oil viscosity increased, oil recovery factor decreased significantly. For VTV method, the oil recovery factor decreased from 20% to 3% by increasing 3.25 times oil viscosity (260 cP to 850 cP) during 30 years of simulation in a nine-spot scheme and using a rectangular reservoir model. For TTHW method in the same situation, the oil recovery factor decreased from 24% to 10%. Both scenarios considered isothermal conditions between injected fluid and original oil in place.

Likewise, **Cunha et al. (2013)** obtained higher recovery factors by increasing water injection flow rate 2.5 times the initial value, but the recovery factor did not increase proportionally, it was kept almost constant. For 0.1 kg/s, 0.2 kg/s, and 0.25 kg/s water-injection flow rates, the oil recovery factor values were 12%, 13%, and 14%, respectively. The oil viscosity was kept constant at 170 cP (intermediate/heavy oil) for all cases and there was no heat transfer in the system.

Costa et al. (2009) analyzed the presence of fracture and injectivity losses while injecting water in a rectangular reservoir model with a five-spot pattern of injection. The comparative parameters were cumulative oil production, cumulative produced water, cumulative injected water, and net present value. These parameters presented different results as oil viscosity changed. In general, oil production was significantly lower for heavy oil if compared to light oil. No thermal conditions were defined and a little variation of water viscosity was considered by differing water surface viscosity and water bottom viscosity.

Alajmi et al. (2009) conducted some experiments with heavy oil obtained from a Middle Eastern reservoir. From a core sample previously flooded with 500 cP oil, water was pumped into the core in different conditions. The most efficient case found for oil recovery was a two-step injection of unheated water and hot water. First, unheated water was injected 3.0 times the pore volume and, then, hot water 1.0 time the pore volume. The final results led to an oil recovery percentage of about 50%. Furthermore, results indicate that to improve oil recovery considerably the volume of injected hot water needs to be higher than 0.5 PVI (Pore Volume Injected) while the optimum hot water volume is at 1.0 PVI.

Goodyear et al. (1996) compared the injection process of hot water and cold water (same temperature of the reservoir). Considering high permeability (1 – 10 Darcy) and oil viscosity of 400 cP, the best scenario for oil recovery ended up in an incremental oil quantity of 18% of original oil-in-place. Moreover, a better economic assessment was observed in the results for water flooding in each individual well by a factor of two.

Harmsen (1971) compared hot-water injection to steam drive and concluded that, even though steam drive is a more stable process, the hot-water injection is cheaper and preferable in some cases. For example, in cases of heavy-oil reservoirs with low viscosity range, up to a few hundred centipoises, or with large well distances the hot water can offer a better efficiency than the steam drive.

All authors indicated that an alternative way for improving efficiency of fluid injection in oil reservoirs, for heavy oil cases, may be heat transfer from injected fluid to oil in place. Consequently, a change in oil viscosity would lead the production process to higher values of oil production and oil displacement in the reservoir.

2. METHODOLOGY

This problem was divided into four parts: (a) Physical Model, (b) Mathematical Model, (c) Simulation Conditions, and (d) Case studies.

2.1 Physical model

The reservoir proposed for this study is hypothetical and arbitrary. Its purpose is merely for testing case studies and comparing them. The physical dimensions of the reservoir model are described in Figure 1 together with the wells positions (Table 1). The scheme injection is staggered line drive with one injection well and four production wells distributed along the reservoir.

Table 1. Coordinate wells positions.

Well	(X [m], Z [m])
Injection	(0, 0)
Production (P1)	(90, -100)
Production (P2)	(90, 100)
Production (P3)	(-90, 100)
Production (P4)	(-90, -100)

Geometry and mesh of this model were built by using Ansys ICEM CFD^{TM} software. Mesh elements are basically hexahedral all over the reservoir. The only refined mesh parts are around the wells' borders due to the proximity of the water flow that comes from the well's injector surface. As a matter of fact, all meshes were composed of 382,080 hexahedral elements and 426,140 nodes. They are also structured all over the reservoir geometry. The refinement around the wellbore can be seen in Figure 2. The measurements for well diameter and well section length are 0.2 m and 5 m, respectively. No mesh test was run before the simulation,

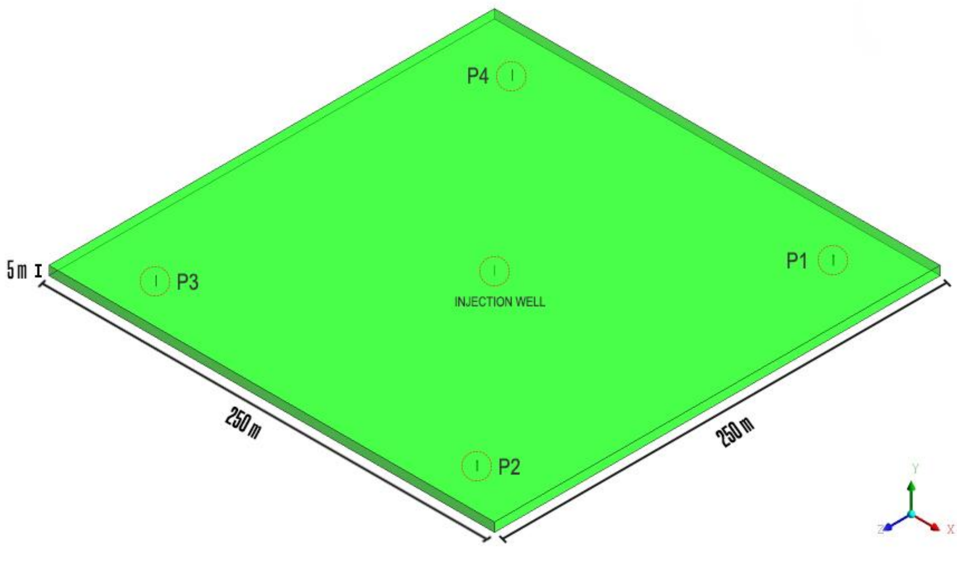

Figure 1. Hypothetical reservoir section with a staggered line injection scheme.

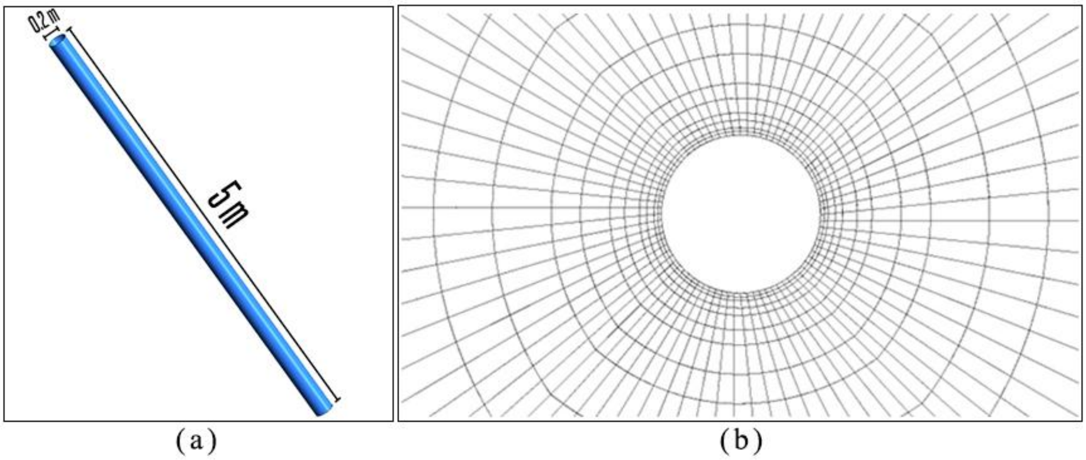

Figure 2. (a) Well geometry (length = 5 m and Radius = 0.2 m) model of the wells and (b) Mesh refinement around the wells for both views, top and bottom.

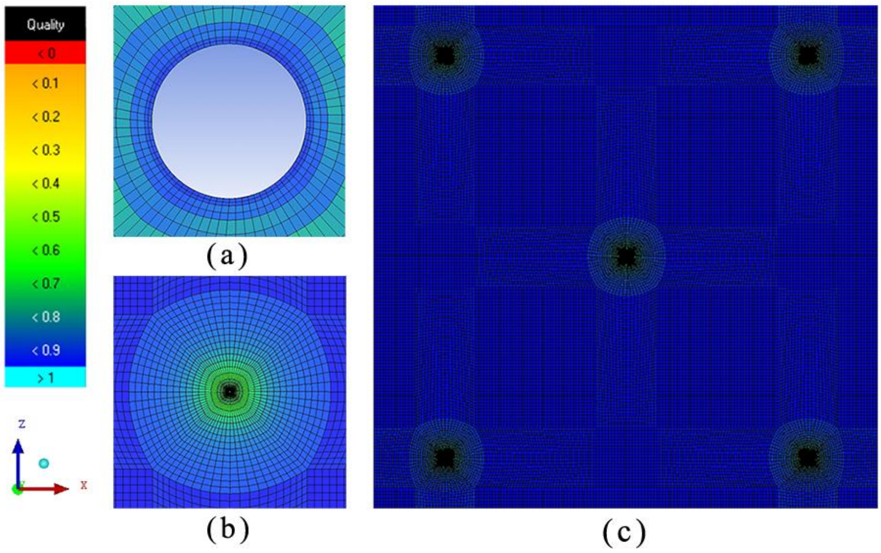

Figure 3. (a) Mesh quality at the well boundaries, (b) mesh quality around the well, and (c) mesh quality across the whole reservoir.

however, the created mesh presented good quality throughout the reservoir and wells as Figure 3 shows. The simulations were run in a computer with the following hardware settings: Processor Intel Core i5-4690K 3.5 GHz (x64-based processor), 16GB of RAM and 1TB of HD storage. The total simulation time for all cases was about 443 hours.

2.2 Mathematical modeling

The software – Ansys CFX 14.0 – utilized to run the case studies in this article solved the principle equations of fluid dynamics and heat transfer: continuity equation, momentum equation, and total energy equation (applied only to the thermal case). The general characteristics of the adopted model are:

- Porous Medium is homogenous, isotropic, and presents no failures;
- Reservoir must be gas-free (dead oil) and has only oil initially;

- Rock properties are constant;
- No chemical reactions;
- No fluid influx or heat transfer from reservoir walls;
- Buoyancy effect is considered all over the reservoir;
- Water and oil viscosities vary according to their respective correlations;
- Flow regime is transient.

There will only be heat transfers between water and oil for the thermal cases, since fluids are submitted to different initial temperatures.

Differently from the black-oil model, for example, this model does not consider gas in solution or vaporized oil in gas; there is no mixing between phases or miscibility. There are just water and oil segregated by and connected via an interface. Each fluid has its own field and for each time interval there is one solution field for each separate phase. Mathematically, density functions in black-oil model, for example, include oil and gas to estimate the density in regions that contain both. The model proposed by this paper does not mix properties to find an intermediate solution, but, rather, it considers fluids individually.

2.2.1 Continuity equation

The continuity equation (Eq. 1), also known as the conservation of mass equation, is expressed as the transfer of liquid mass to or from a control volume during a time period Δt that is equal to the mass liquid variation inside the volume control during the same Δt.

$$\frac{\partial}{\partial t}(\phi\rho) + \nabla\cdot\left(\rho\vec{K}\vec{U}\right) = 0 \tag{1}$$

Where:

$\rho \rightarrow$ Density

$\vec{U} \rightarrow$ True velocity vector

$\phi \rightarrow$ Homogenous porosity

$\vec{K} \rightarrow$ Area porosity tensor

2.2.2 Momentum equation

This equation states that the total forces acting on a control volume are equal to a rate in which the momentum changes inside the control volume plus the rate in which the momentum flows to outside of the control volume, subtracted from the rate in which the momentum flows to inside of the control volume. Choosing an infinitesimal control volume and balancing the momentum in all directions it is possible to have Eq. (2).

$$\begin{aligned}&\frac{\partial}{\partial t}(\phi\rho\vec{U}) + \nabla\cdot\left(\rho\left(\vec{K}\vec{U}\right)\otimes\vec{U}\right) - \\ &-\nabla\left(\mu_e\vec{K}\left(\nabla\vec{U} + \left(\nabla\vec{U}\right)^T - \frac{2}{3}\delta\nabla\vec{U}\right)\right) = \\ &= \phi\vec{S}_M - \phi\nabla p\end{aligned} \tag{2}$$

Where:

$\rho \rightarrow$ Density

$\vec{U} \rightarrow$ True velocity vector

$\phi \rightarrow$ Homogenous porosity

$\vec{S}_M \rightarrow$ User specified mass sources

$\mu_e \rightarrow$ Effective viscosity

$\vec{K} \rightarrow$ Area porosity tensor

$p \rightarrow$ Pressure vector

$\delta \rightarrow$ Identity matrix

2.2.3 Total energy equation

The total thermal energy balance in an infinitesimal control volume can be obtained by following the process: (Inlet heat transfer rate) – (Outlet heat transfer rate) + (rate of heat generation inside control volume) = (rate of energy change of infinitesimal element). From this balance, the following equation (Eq. 3) is found:

$$\begin{aligned}&\frac{\partial}{\partial t}(\phi\rho H) + \nabla\cdot\left(\rho\vec{K}\vec{U}H\right) = \\ &= \nabla\cdot\left(\Gamma_e\vec{K}\nabla H\right) + \phi S_k^H + Q_{fs}\end{aligned} \tag{3}$$

Where:

$\rho \rightarrow$ Density

ϕ → Homogenous porosity

$\vec{U}$ → True velocity vector

$\vec{S}_k^H$ → A vector that describes specified heat source

$\vec{K}$ → Area porosity tensor

H → Enthalpy

Γ_e → Effective diffusivity

Q_{fs} → Interfacial heat transfer between the fluid and the solid

There are still some equations that consider important parameters to this model. One of them, for example, is the equation for momentum loss through a porous isotropic region found in the momentum equation. It is represented by the components below:

$$S_{M,x} = -\frac{\mu}{K_{perm}} U_x - K_{loss} \frac{\rho}{2} |\vec{U}| U_x \quad (4)$$

$$S_{M,y} = -\frac{\mu}{K_{perm}} U_y - K_{loss} \frac{\rho}{2} |\vec{U}| U_y \quad (5)$$

$$S_{M,z} = -\frac{\mu}{K_{perm}} U_z - K_{loss} \frac{\rho}{2} |\vec{U}| U_z \quad (6)$$

Where K_{perm} is permeability and K_{loss} is quadratic loss coefficient. The linear component of this source represents viscous losses and the quadratic term represents inertial losses **(CFX 14.0 Guide, 2011)**.

2.3 Simulation conditions

The general assumptions all over the reservoir for the application of the case studies are:

- Oil Saturation (S_o) is 1 for the whole reservoir and production well initially;
- Water Saturation (S_w) is 1 for the injection wells initially;
- Initial reservoir pressure is 3441.9 psi;
- Initial static pressure for production wells is 2000 psi;
- There is no slipping condition for the walls, bottom and top of the reservoir, i.e. fluid velocity is null at these points;
- Water injection flow rate is 0.3 kg/s;
- Flowing pattern is laminar and there is a dependence between fluids and their densities;
- Reservoir permeability (k) is $1\,x\,10^{-13}\,m^2$ and porosity (ϕ) is 20%;
- Isothermal case considers temperature as 233.24°F everywhere. Whereas, thermal case considers injected water at 283.24°F;
- Nusselt number is equal to 2 for thermal cases;
- The total time of simulation was 35,040 hours (4 years) and timesteps were equal to 2 hours;
- Adopted convergence criteria was RMS with a target of $1x10^{-4}$ and maximum loops for interaction was 10.

2.4 Case studies

Four different cases are proposed for this study. The main objective is to observe how some parameters behave with or without heat transfer. Therefore, different initial conditions and scenarios are set to each case (see Tables 2 to 5).

Fluids viscosities vary according to **Beal (1946)** and **Dutra Junior (1987)** correlations. Beal's correlation is limited to atmospheric pressure and temperature at or above 60°F, moreover the oil must be gas-free (dead oil). However, in this paper there is an extrapolation of pressure since the oil is under reservoir pressure rather than atmospheric pressure. Figure 4 shows how distinctly oils with

Table 2. Characteristics for each case study.

Case	Method	Oil	Initial Temp. (°F)	
			Water	Reservoir
I	Isothermal	11 °API	233.24	233.24
II	Isothermal	17 °API	233.24	233.24
III	Thermal	11 °API	283.24	233.24
IV	Thermal	17 °API	283.24	233.24

Table 3. Fluids characteristics for cases I and III.

Properties	Petroleum	Water
Density (kg/m^3)	989	997
Molar mass (kg/mol)	105.47	18.02
Viscosity (Pa.s)	Beal (1946)	Dutra Junior (1987)

Table 4. Fluids characteristics for cases II and IV.

Properties	Petroleum	Water
Density (kg/m^3)	951	997
Molar mass (kg/mol)	105.47	18.02
Viscosity (Pa.s)	Beal (1946)	Dutra Junior (1987)

Table 5. Beal (1946) and **Dutra Junior (1987)** equations for viscosity.

$$\mu_{oil} = \left(0.32 + \frac{1.8(10^7)}{°API^{4.53}}\right)\left(\frac{360}{T + 200}\right)^{\exp\left(2.302585\left(0.43+\frac{8.33}{°API}\right)\right)} \tag{7}$$

$$\mu_w = \left(\frac{2.185}{(40.12 + 0.0051547(T))T - 1000}\right)(1000) \tag{8}$$

Where: $\mu_{oil} \rightarrow$ Oil viscosity (cP); $\mu_w \rightarrow$ Water viscosity (cP); $T \rightarrow$ Temperature (°F)

different API gravity behave as temperature varies. Likewise, water has a variation but it is negligible between 100°F and 300°F. To analyze particularly only the effect of viscosity variation due to heat transfer, densities were kept constant despite temperature variation.

3. RESULTS AND DISCUSSIONS

The conventional oil recovery method used in this simulation is the continuous water injection in a square reservoir analogue to a prism having square bases. Searching for comparison among the four cases described in previous section, the simulation reached 35,040 hours (4 years) of continuous water injection for all cases. The main

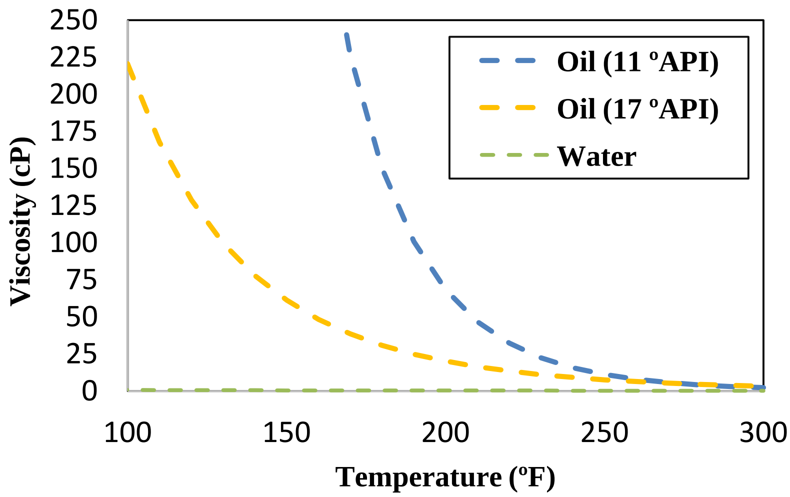

Figure 4. Viscosity variation for **Beal (1946)** and **Dutra Junior (1987)** equations.

parameters submitted to analysis were: Oil flow rate, water flow rate, injection pressure, produced water volume, recovery factor and injected pore volume. All these parameters were analyzed together in this section.

3.1 Qualitative results

Firstly, Figure 5 shows water flowing from the injection well to production wells directions. Although, this figure refers to case I, other cases behave similarly since there is a radial flowing pattern coming out from the center of the reservoir. From Figures 5 to 8, it is easily noticeable that, for having a higher density, water goes down to the bottom of the reservoir and follows the easiest way toward production wells. These ways or thin and long water veins are called fingers. Fingers or streamers are a result of unstable oil-water interface **(van Meurs and van der Poel, 1958)**, for they have significantly different viscosities. Consequently, they cause an early

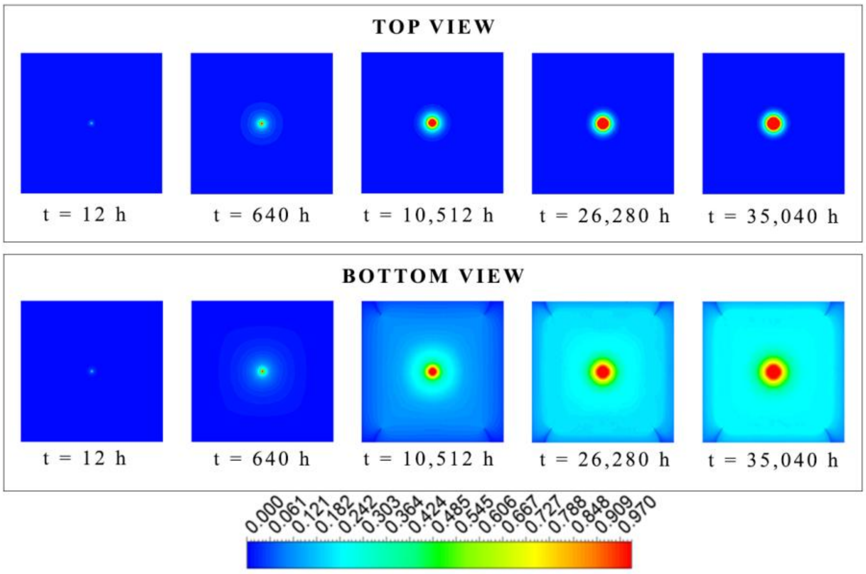

Figure 5. Water volume fractions at top and bottom of the reservoir for case I.

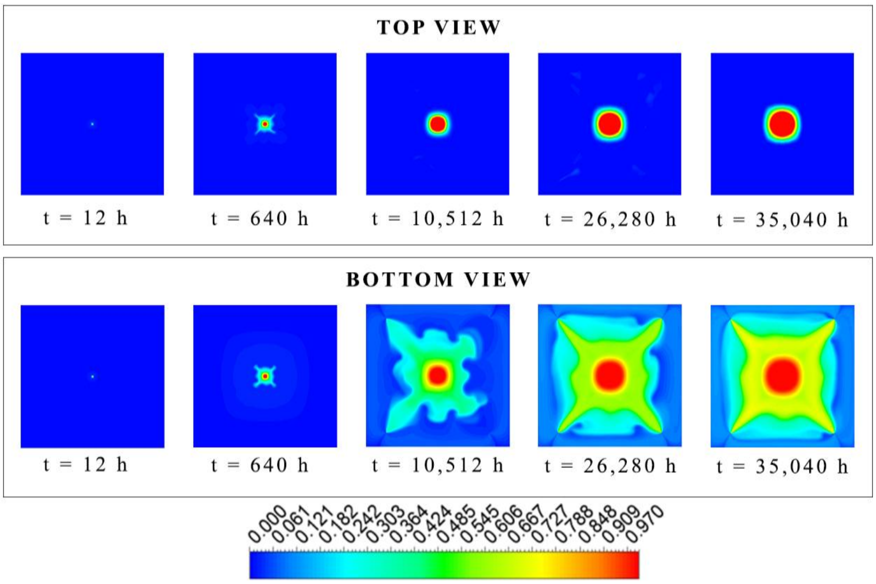

Figure 6. Water volume fractions at top and bottom of the reservoir for case III.

breakthrough (moment which water is produced by production wells) and low sweep efficiency to the water injection process (**Araktingi & Orr Jr, 1993**). Comparing qualitatively water fraction distribution over the reservoir for cases I and III, it is noticeable a wider presence of water-oil bulk. It likely happens due to the decreasing of oil viscosity led by heat transfer from water to oil. Consequently, more oil is heated and produced by the production wells, for it reaches them faster.

Figures 5 and 6 show distinctively a massive area reached by water that points out to the four production wells. This difference is seen clearly from 640 hours of water injection either on top view or bottom view. At 10,512 hours, water has already gotten further towards production wells. Together, the extension of oil-water bulk is higher for case III than for case I, inferring a higher oil mobility since oil viscosity is decreased by heat received from water. Likewise, Figures 7 and 8 confirm that a bigger vertical area was swept by water. It is still possible to see less fingers close to production wells for times over 10,512 hours. A similar behavior should be found for the cases II and IV, but with a less significant difference between isothermal and thermal cases, for the oil degree API is higher.

3.2 Quantitative results

3.2.1 Flow rates

Figure 9 reveals a different behavior for each case but, somehow, they show a similar pattern. For all cases, the breakthrough time is practically

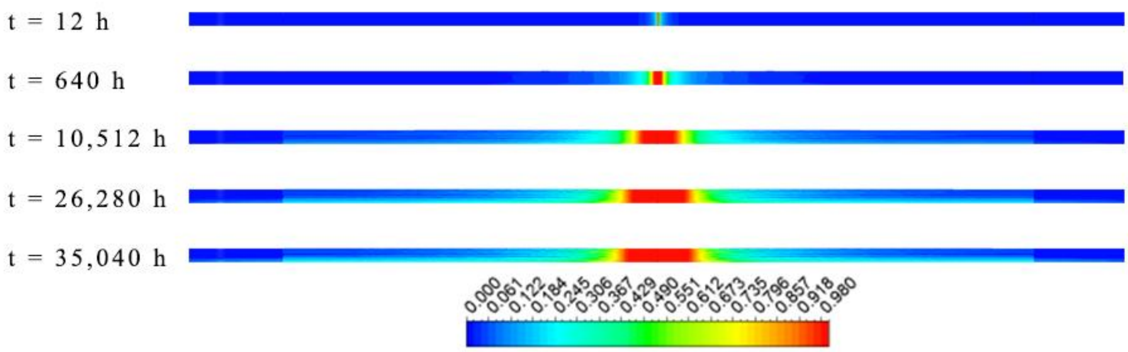

Figure 7. Water volume fractions from a perspective of an orthogonal plane through two production wells and the injection well for case I (11°API oil – Isothermal).

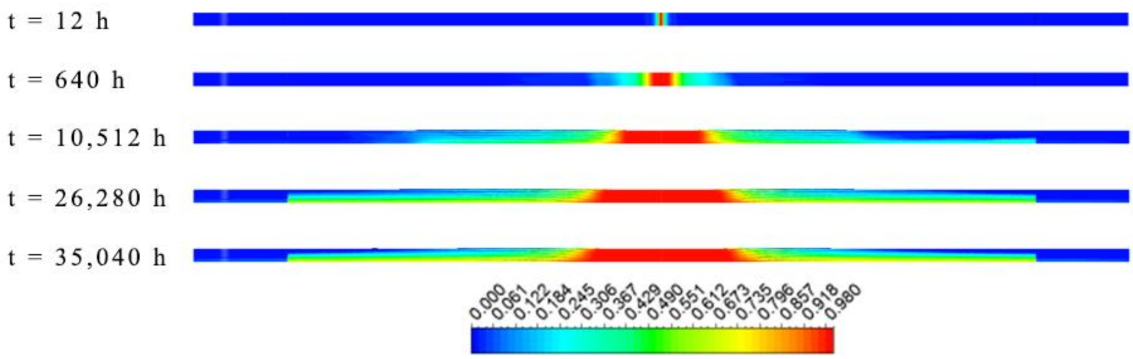

Figure 8. Water volume fractions from a perspective of an orthogonal plane through two production wells and the injection well for case III (11°API oil – Thermal).

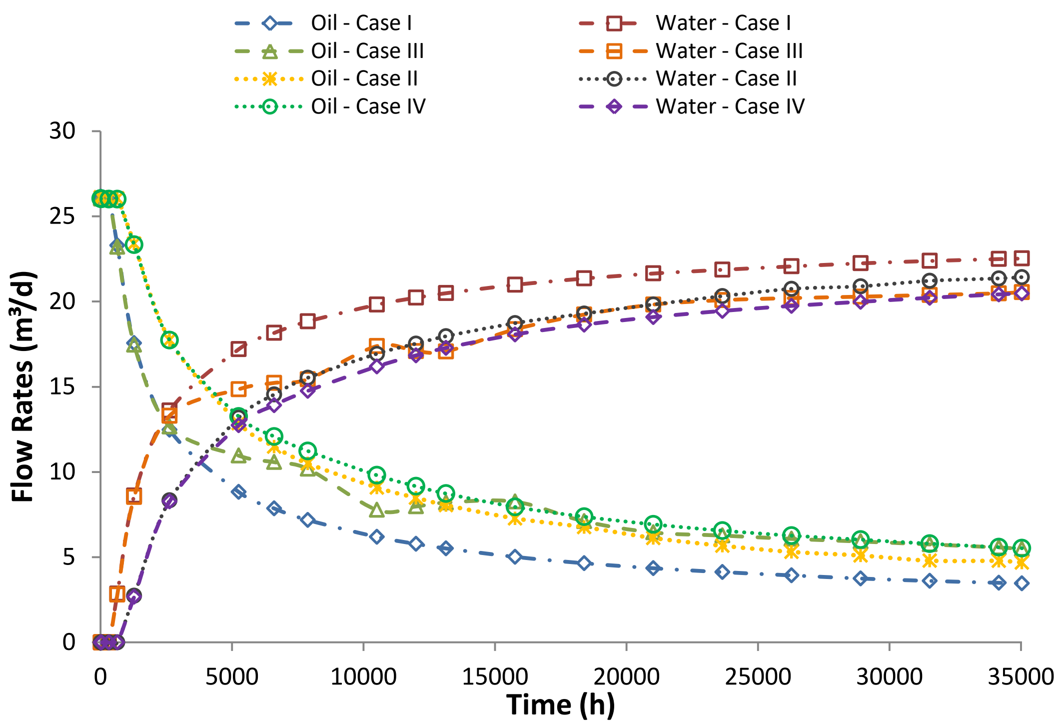

Figure 9. Flow rates of all four production wells versus time.

the same. The first easily perceptible difference is between cases I/III and II/IV, because water flow rate and oil flow rate curves cross each other at different times, implying a slower decreasing oil production through time. While cases I and III have water and oil curves crossed at approximately 2,500 hours, cases II and IV have a cross point at 5,256 hours. The main reason for this is oil density differences, while API gravity is 11 for cases I and III, for cases II and IV it is 17, i. e. density has an impact over oil and water displacement.

Comparing cases I and III, one can see the same behavior of flow rate curves before the cross point. After this point, curves are different. As heat transfer occurs in case III, oil mobility increases and oil flow rate becomes higher than its value for case I. Similarly, water flow rate decreases when compared to case I. Analogously, same pattern is found by comparing cases II and IV, but the difference between them is lower.

3.2.2 Injection pressure

Pressure drops differently for each case, but curves behaviors are similar (Figure 10). Cases with different oil density have different starting points. Expectedly, pressure goes down in injection well, for water is pumped into the reservoir and produced by production wells. As production wells are at 2000 psi and reservoir pressure drops gradually, injection pressure is also affected and tends to stabilize close to production wells pressure. Numerically, pressure drop percentages are 36.21%, 37.31%, 24%, and 24.41% for cases I, II, III, and IV, respectively. Clearly, pressure drop is considerably lower for thermal cases, for water temperature is higher and oil in reservoir reduces its viscosity (flowing resistance) by heating it up.

Thermal cases suggest that less pump pressure is needed during injection water process, which gives it an advantage over isothermal method for operational and economic reasons.

3.2.3 Water volume

Figure 11 contrasts total water volume injected and total produced water. Firstly, it is perceptible that thermal methods produce less water than isothermal methods which means that more water is accumulated and dispersed all over the reservoir. Consequently, more oil is produced by production wells. Comparing cases I and III at final simulation

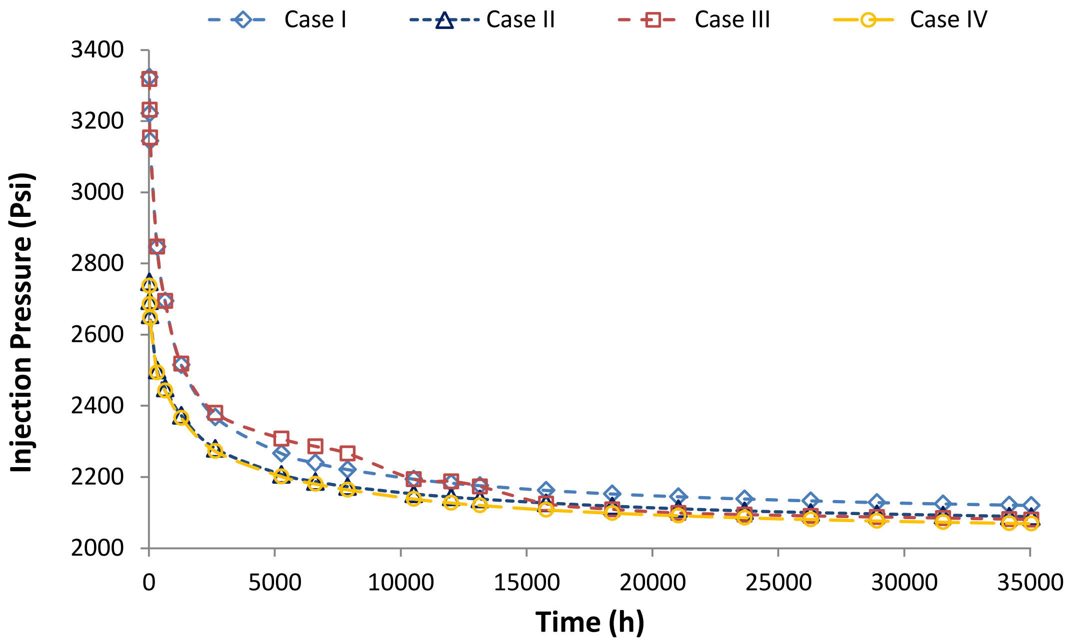

Figure 10. Injection pressure in the injection well versus time.

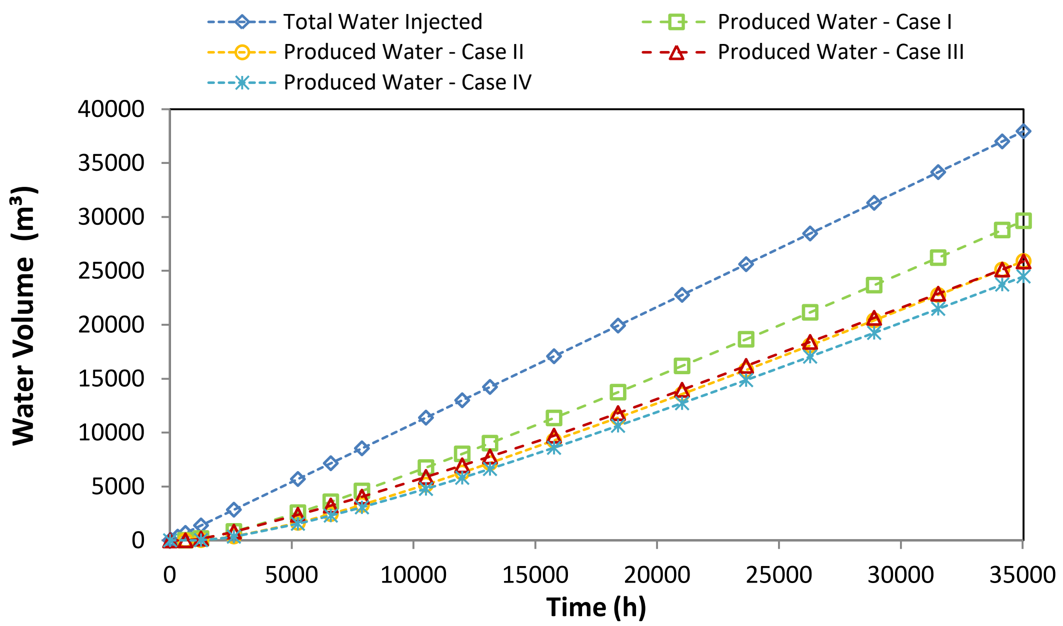

Figure 11. Water volumes versus time.

time, case III produced 12.74% less water than case I. The same happens with cases II and IV, case IV produced 5.5% less water compared to case II. This means that thermal method has more impact for heavier oils.

3.2.4 Recovery factor

As an important term, recovery factor is the percentage from the relation between produced oil and original oil in place. From Figure 12 and 13, values of recovery factor vary with time and injected pore volume, respectively.

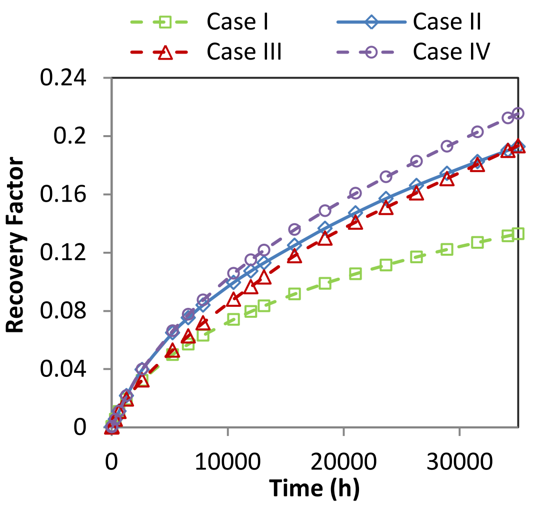

Figure 12. Recovery factor for all proposed cases over time.

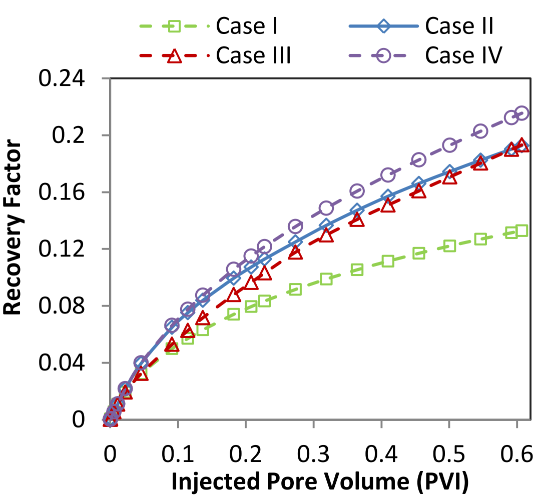

Figure 13. Recovery factor versus PVI.

The main ways of comparison are between different method cases and between different oil densities. The first way leads to a difference of 6% between cases I and III, and 2.28% between cases II and IV. Secondly, comparing both isothermal cases (I and II), a difference is 6% against 2.22% from thermal cases (III and IV).

The best-case scenario, comparison between case I and III, found an improvement of 31.26% over isothermal method. For the oil with API gravity close to medium oil (17 °API) classification, the improvement reached 10.58% (cases II and IV).

Based on **Alajmi et al.'s (2009)** conclusions, the optimum injected hot water volume would be about 1.0 PVI. Even with the maximum of 0.6 PVI during simulation, a significant improvement was accomplished at the final injection time. So, there is still potential to get more than 31.26% of improvement between isothermal and thermal methods for heavy oils.

In general, the model developed in this paper can provide the main important parameters such as oil and water rate over time, cumulative production from wells, percent recovery over time, water breakthrough in production wells, water-oil ratio over time, reservoir pressure variation over time, saturation and distribution of fluids in the reservoir, changes in bottom hole pressure and well productivity index. However, this model fails to reveal other parameters, for example, economic life of wells and reservoir, fluid influx through reservoir boundaries, residual oil saturation, presence of gas and compositional phases of oil separated by their carbon composition. Also, this model does not consider any historical data matching or complex geological rock composition. Its purpose focuses mainly on assessing the impact of heat transfer on heavy oil during water injection process.

4. CONCLUSIONS

The hot water injection method presented itself as an advantageous opportunity for producing more oil than just injecting water at the same temperature of reservoir into rock. Although this study does not show the full potential of thermal method by heating up oil in place, it is sufficient to show how it works better in scenarios which heavy oil is considered. After a simulation of 35,040 hours, hot water injection with a variation of 50ºF between reservoir temperature and injected water reached a result of 19.33% recovery factor for an 11ºAPI oil-type which represented an improvement of 31.26% over a case with no temperature variation between reservoir and injected water. Submitting a 17ºAPI oil-type to the same conditions, the improvement reached just 10.58%, even though recovery factor got 2% more to thermal method than isothermal method. Thus, it is conclusive that thermal methods work well for heavy oils in comparison to light oils.

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